# Exercise 34

Find the limit or show that it does not exist.

$$\lim_{x\to-\infty}\frac{1+x^6}{x^4+1}$$

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### Solution

Make the substitution, $u=-x$, so that as $x\to-\infty$, $u\to\infty$. Then multiply the numerator and denominator by the reciprocal of the highest power of $u$ in the denominator.

$$\begin{aligned}
\lim_{x\to-\infty}\frac{1+x^6}{x^4+1} &= \lim_{u\to\infty}\frac{1+(-u)^6}{(-u)^4+1} \\
&= \lim_{u\to\infty}\frac{1+u^6}{u^4+1} \\
&= \lim_{u\to\infty}\frac{1+u^6}{u^4+1}\cdot\frac{\frac{1}{u^4}}{\frac{1}{u^4}} \\
&= \lim_{u\to\infty}\frac{(1+u^6)\frac{1}{u^4}}{(u^4+1)\frac{1}{u^4}} \\
&= \lim_{u\to\infty}\frac{\frac{1}{u^4}+u^2}{1+\frac{1}{u^4}} \\
&= \frac{\lim\limits_{u\to\infty}\left(\frac{1}{u^4}+u^2\right)}{\lim\limits_{u\to\infty}\left(1+\frac{1}{u^4}\right)} \\
&= \frac{\lim\limits_{u\to\infty}\frac{1}{u^4}+\lim\limits_{u\to\infty}u^2}{\lim\limits_{u\to\infty}1+\lim\limits_{u\to\infty}\frac{1}{u^4}} \\
&= \frac{0+\infty}{1+0} \\
&= \infty
\end{aligned}$$